POEMA

H2020-MSCA-ITN-2018

Polynomial Optimization, Efficiency through Moments and Algebra

PERSONAL CAREER DEVELOPMENT PLAN

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ESR Name:	Vu Trung Hieu
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Advisors:	Victor Magron, Mohab Safey El Din
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Table of Contents

Introduction

The Personal Career Development Plan (PCDP) describes both near and long term objectives of the fellow, to reflect on their progress, plan their future development, and take actions to realize their plans. The document must be completed and updated every 12 month by the fellow and his/her advisor. It will be monitored yearly by the Educational Committee who will also provide the feedback assessment results of the training programme on the occasion of the yearly meeting. Major deviations from the plan should be reported to the Educational Committee.

1 Individual Research Plan

1.1 Host Institution

Sorbonne University, Paris, France

1.2 PhD Advisor(s)

Victor Magron, LAAS, Toulouse, France
Mohab Safey El Din, Sorbonne University, Paris, France

1.3 PhD Thesis Supervisor Committee (if applicable)

1.4 Short overall project description

Title: Exact certificates of positivity for polynomial optimization

This PhD project is in between computer algebra, real algebraic geometry, real algebra, and polynomial optimization. A popular strategy to prove lower bounds on polynomial optimization problems is to write non-negative polynomials as sums-of-squares. One issue is that not all non-negative polynomials are sums-of-squares. A maybe more important problem is that there are non-negative polynomials which are sums of squares of polynomials with real coefficients but not sums of squares with rational coefficients. This is a lock for computing exact certificates of non-negativity.

This PhD project is based on the following idea. One can add constraints to our initial polynomial optimization problem without modifying the solution of the problem we have at hand. This allows us to reduce the dimension of the space we are considering. Algebraically, this will allow us to reduce the number of variables. In favourable cases, we expect to reduce our problems to univariate ones for which non-negative polynomials with rational coefficients are always sums of squares of polynomials with rational coefficients.

We will develop this strategy, take into account the structures that appear in applications and implement the algorithms we will obtain.

1.5 First secondment

University of Konstanz: from 02/2021 to 04/2021, online

1.6 Second secondment

RTE France: from 02/2022 to 04/2022

2 Research Outputs, Dissemination and Mobility

2.1 Research results

1/ Victor Magron, Mohab Safey El Din, Trung-Hieu Vu; *Sum of squares decompositions of polynomials over their gradient ideals with rational coefficients*

Decomposing a multivariate polynomial with rational coefficients as a sum of squares (SOS) of polynomials or rational functions provides a certificate for its nonnegativity. This task, which is important for certified optimization, is hard in general because there exist nonnegative polynomials that are not SOS, and even if they are, there is no guarantee that there exists an SOS decomposition involving rational coefficients. Parrilo proposed a nonnegativity certificate by decomposing a polynomial as an SOS of polynomials modulo its gradient ideal with real coefficients under certain assumptions.

Given a nonnegative multivariate polynomial f, where the coefficients are rational and its gradient ideal is zero-dimensional radical, we present two algorithms computing decompositions that are SOS of polynomials or rational functions of f modulo its gradient ideal with rational coefficients. For examples which are out of reach with direct SOS methods, we show that our algorithms can certify nonnegativity. Furthermore, we provide bit complexity estimates for these algorithms' runtimes, which are exponential in the number of variables, polynomial in the degree, and quadratic in the maximum bitsize of f. The algorithms have been implemented in Maple. We showed practical experiments to illustrate the behavior of our algorithms.

2/ Trung-Hieu Vu; *On the solution existence and stability of polynomial optimization problems*

We introduce and investigate a regularity condition in the asymptotic sense for the optimization problems whose objective functions are polynomial. The normalization argument in asymptotic analysis enables us to see the solution existence as well as the solution stability of these problems. We prove a Frank-Wolfe type theorem for regular problems and an Eaves type theorem for non-regular pseudoconvex optimization problems. Under the regularity condition, we prove several local properties of the solution map of polynomial optimization problems such as local boundedness, upper semicontinuity, and local upper-Holder stability.

2.2 Research publications

We completed a joint paper entitled "*Sum of squares decompositions of polynomials over their gradient ideals with rational coefficients"* based on the obtained results. We will submit the paper for publication in April 2021.

The paper *"On the solution existence and stability of polynomial optimization problems"* has been submitted for publication in Optimization Letters.

2.3 Dissemination and networking

I attended the following online scientific conferences:

Within the POEMA network:
1/ POEMA ESR Day, October 16, 2020
2/ POEMA 2nd workshop, October 20 - December 11, 2020
3/ POEMA 3rd workshop, January 29 - February 17, 2021

Outside of the POEMA network:
4/ The International Symposium on Symbolic and Algebraic Computation (ISSAC), July 20 – 23, 2020
5/ French Computer Algebra Days, March 1 - 5, 2021

I gave two talks in POEMA ESR Day (1) and French Computer Algebra Days (5).

I am doing online the first secondment at the University of Konstanz, from 02/2021 to 04/2021.

2.4 Software, Data, other

We have two software packages corresponding the two algorithms but they have not shared at the moment.

3 Personal Training Plan

3.1 Scientific training courses

I attended the following online scientific training courses:

1/ POEMA 1st Learning Weeks, May 27 – September 16, 2020

I plan to attend the following scientific training courses:

2/ General online Julia training, April 16, 2021

3/ POEMA Learning Week 2, June 21 – 25, 2021

I have been attending the weekly joint seminar of the PolSys team (LIP6, Sorbonne University) and the Specfun team (INRIA Saclay) since February 2020.

I have attended the weekly reading seminar on Faugere's Algorithm organized by PolSys team at LIP6, Sorbonne University, from February to April 2020.

I have attended a course given by Prof. Mohab Safey El Din and Dr. Jérémy Berthomieu on algorithms in computer algebra for PhD students in the PolSys team, from March to June 2020.

My advisors and I have been meeting and discussing weekly on research topics. I have been learning computer algebra, real algebraic geometry, and polynomial optimization from these meetings.

3.2 Complementary training courses

I have leaned the following ones from the advisors and collegues: Skills in writing mathematical papers; Skills in writing slides and giving presentation in a conference.

3.3 Professional skill development

Learn Maple and Julia software to implement algorithms in my research. Improve skills in writing mathematical papers. Continue to improve English and French.

4 Personal Career Development

4.1 Plan for the next period

- Develop the obtained results for constrained cases
- Study on sums-of-squares decompositions of positive complex univariate polynomials
- Study polynomial optimization problems with some algebraic structure such as sparsity, weighted/multi-homogeneity and invariance by the action of a finite group
- Study sparse polynomial optimization problems and "sparse" variant Polya's theorem and its consequences in polynomial optimization

4.2 Career objectives (Postdoctoral project, …)

- Write research articles that will be submitted to the top conferences/journals
- Design algorithms and implement a software package which is usable by the scientific community
- Write and defend successfully my PhD thesis
- Apply a post-doctoral position on polynomial optimization, computer algebra and/or real algebraic geometry