



POEMA

H2020-MSCA-ITN-2018

**Polynomial Optimization, Efficiency through
Moments and Algebra**

PERSONAL CAREER DEVELOPMENT PLAN

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Table of Contents

Introduction	5
1 Individual Research Plan	6
1.1 Host Institution	6
1.2 PhD Advisor(s)	6
1.3 PhD Thesis Supervisor Committee (if applicable)	6
1.4 Short overall project description	6
1.5 First secondment	6
1.6 Second secondment	6
2 Research Outputs, Dissemination and Mobility	7
2.1 Research results	7
2.2 Research publications	7
2.3 Dissemination and networking	7
2.4 Software, Data, other	7
3 Personal Training Plan	8
3.1 Scientific training courses	8
3.2 Complementary training courses	8
3.3 Professional skill development	8
4 Personal Career Development	9
4.1 Plan for the next period	9
4.2 Career objectives (Postdoctoral project, ...)	9

Introduction

The Personal Career Development Plan (PCDP) describes both near and long term objectives of the fellow, to reflect on their progress, plan their future development, and take actions to realize their plans. The document must be completed and updated every 12 month by the fellow and his/her advisor. It will be monitored yearly by the Educational Committee who will also provide the feedback assessment results of the training programme on the occasion of the yearly meeting. Major deviations from the plan should be reported to the Educational Committee.

1 Individual Research Plan

1.1 Host Institution

INRIA [Méditerranée](#)

1.2 PhD Advisor(s)

HUBERT, Evelyne

1.3 PhD Thesis Supervisor Committee (if applicable)

1.4 Short overall project description

This project aims at improving the robustness of moment methods for polynomial optimization. Based on group theoretic techniques we shall develop new algorithms for algebraic computations in non monomial bases. Relaxation methods have seen the size of the moment matrices decrease by intertwining some steps of polynomial reductions, a technique typical of symbolic computation, with steps of semi-definite programming, performed numerically. Monomial bases have long prevailed in symbolic computation and polynomial reduction techniques are essentially based on monomial rewriting. Yet monomials need not form an appropriate basis. On one hand one cannot preserve symmetry with these. On the other hand, in the univariate case, analysts have stayed clear of these bases and used orthogonal polynomials for their good numerical properties. Like other special functions, multivariate orthogonal polynomials have strong ties to group representations. This has hardly been explored algorithmically, neither in numerical nor symbolic computation. We shall exploit the group theoretic foundations of multivariate Chebyshev polynomials to establish efficient and robust arithmetics in forming moment matrices and polynomial reductions. In the presence of symmetry, also given in terms of a group, adapted bases allow to block diagonalize the moment matrices, thus splitting the original problem in smaller pieces, improving the overall complexity and giving a better chance at numerical stability. We shall provide new technologies to compute and work with symmetry adapted bases efficiently.

1.5 First secondment

11/2020 – 01/2021 at UiT Tromsø with Cordian Riener (COMPLETED)

1.6 Second secondment

08/2021 – 10/2021 at NAG Oxford (date may change)

2 Research Outputs, Dissemination and Mobility

2.1 Research results

During the first year, my supervisor and I studied a family of polynomial, known as the generalized Chebyshev polynomials, which originate from root systems and Weyl groups. We proved several properties such as orthogonality, behavior of the zeros and periodicity.

During the secondment at UiT, we applied our results to compute a bound for the chromatic number of a lattice. We are currently preparing the results to be published.

As a side project, I worked with Viktor Levandovskyy and Karim Abou Zeid, with whom I wrote my Master thesis, on the computation of Groebner bases over free non-commutative polynomial algebras with coefficients in rings.

2.2 Research publications

- *“Computation of free non-commutative Gröbner Bases over Z with Singular:Letterplace” with Viktor Levandovskyy and Karim Abou Zeid (conference article at ISSAC 2020)*
<https://hal.archives-ouvertes.fr/hal-02496535v2>
- *“Computing Free Non-commutative Groebner Bases over Z with Singular:Letterplace “ with Viktor Levandovskyy and Karim Abou Zeid (extension of the above ISSAC paper and preprint submitted to JSC)*
<https://hal.archives-ouvertes.fr/hal-03085431/>

2.3 Dissemination and networking

- 01/15/2020 till 01/17/2020: First POEMA Workshop in Florence - <http://poema-network.eu/index.php/news-and-events/project-workshops/2-poema-1st-workshop>
- 03/11/2020: First POEMA Interim Review with EC - <http://poema-network.eu/index.php/news-and-events/project-workshops/5-1st-interim-review-with-ec>
- 03/23/2020 till 04/03/2020: Second POEMA Workshop in Konstanz - <http://poema-network.eu/index.php/news-and-events/project-workshops/3-poema-learning-week-and-2nd-workshop> (CANCELLED)
- ESR 10 blog - <https://esr10-poema.blogspot.com/>
- Website - <https://www.sop.inria.fr/members/Tobias.Metzlaff/>
- 03/2020: JNCF <https://conferences.cirm-math.fr/2326.html>
- 05 till 09/2020: POEMA Online learning weeks <http://poema-network.eu/index.php/news-and-events/project-workshops/7-poema-online-learning-weeks>
- 06/2020 till 02/2021: SFO <https://gdrmoa.math.cnrs.fr/seminaire-francais-optimisation/>
- 07/2020: ISSAC <https://issac-conference.org/2020/>
- 07/2020: ICMS <http://icms-conference.org/2020/>
- 15/10/2020: POEMA ESR Day <http://poema-network.eu/index.php/news-and-events/project-workshops/8-poema-esr-event>
- 20/10/2020: POEMA Second Workshop <http://poema-network.eu/index.php/news-and-events/project-workshops/6-poema-learning-week-and-2nd-workshop>

- 01/2021 till 02/2021: POEMA Third Workshop <http://poema-network.eu/index.php/news-and-events/project-workshops/9-poema-workshop-3>
- 01/2021: Mørketidens Mattemøte <https://site.uit.no/mmm/programm-20201/>

2.4 Software, Data, other

Provide (and update) a list of materials (data, software,...) that you have produced with the aim of making your research reproducible for other researchers within and/or outside the POEMA network.

3 Personal Training Plan

3.1 Scientific training courses

-Participation at courses/talks/exercises at first POEMA Workshop about Polynomial Optimization, Tensor Decomposition, SOS and MOM, Computational Algebra, Groebner Bases, Interpolation

<http://poema-network.eu/index.php/news-and-events/project-workshops/2-poema-1st-workshop>

3.2 Complementary training courses

I participated in the semiregular INRIA PhD seminars <https://phd-seminars-sam.inria.fr/fr/>.

3.3 Professional skill development

- *Management skills*
- *Communication skills*
- *Technical skills*
- *Additional skills*

4 Personal Career Development

4.1 Plan for the next period

2020:

- Research on Cubature formulae and minimization of number of nodes
- Representation theory: Root systems and Chebyshev polynomials
- Find new symmetry adapted bases
- Non-commutative Groebner bases over principal ideal rings

2021:

- Paper on Chromatic numbers originating from root systems
- Polynomial optimization on deltoids
- Bound for root systems D_n and E_n
- Proof of the Procesi Schwarz theorem for multiplicative group actions

4.2 Career objectives (Postdoctoral project, ...)

- *Career objectives, self-assessment and plans for the next period*