Title: Orbit Spaces for exponential Weyl Group Actions and the Chromatic Number of R^n

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Abstract:

The Weyl group of a crystallographic irreducible reduced root system has an exponential action on the complex torus, which is formalized on the ring of Laurent polynomials. The orbit space of this action is the image of the generators of the ring of invariants. For the four classical types A, B, C and D, we show that the orbit space is a basic semi-algebraic set and present the polynomial inequalities as the locus of positive semi-definiteness of explicit symmetric Hermite matrices. The resulting domain is the region of orthogonality for generalized Chebyshev polynomials, which have connections to topics such as Fourier analysis and representations of Lie algebras. As an application, we consider the problem of computing the chromatic number of R^n for a polytope norm. We show how to rewrite this problem in terms of generalized Chebyshev polynomials and compute a spectral bound as the solution of a polynomial optimization problem on the orbit space of the associated Weyl group.

Based on joint work with Evelyne Hubert, Philippe Moustrou and Cordian Riener.