Lecture: Sparse polynomial optimization

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Abstract: Polynomial optimization is an emerging field extensively developed in the last two decades, growing from the discovery of the Moment-sums of squares (Moment-SOS) hierarchy by Lasserre, which provides numerical certificates for positive polynomials. One key advantage of the hierarchy is the ability to model a wide range of problems, in the static/dynamical and (non)commutative settings, while using optimization formulations, which can be in turn solved with efficient numerical tools.

However, such optimization methods still encompass many major issues on the practical side due to a lack of scalability of the hierarchy in its initial form.

Fortunately, for many applications coming from real-world problems, we can look at the problem in the eyes and exploit the inherent data structure arising from the cost and constraints describing the problem. In this lecture, I will outline how to improve the scalability of the hierarchy by exploiting the specific sparsity structure of the input data.

The first part of the lecture will focus on the notion of "correlative sparsity", occurring when there are few correlations between the variables of the input problem. The second part will present a complementary framework, where we show how to exploit a distinct notion of sparsity, called "term sparsity", occurring when there are a small number of terms involved in the input problem by comparison with the fully dense case. For both frameworks, I will provide several illustrations on important applications arising from various fields, including roundoff error bounds (computer arithmetic), robustness of deep networks (machine learning), noncommutative optimization (entanglement in quantum information), stability of control systems, and AC optimal power-flow (energy networks).