Title: The Lasserre hierarchy for binary polynomial optimization

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Abstract: We consider the hierarchy of sum-of-squares approximations for the problem of computing the minimum value of a polynomial f over the n-dimensional boolean hypercube. This hierarchy provides lower bounds for each order r and is known to be exact at (roughly) order (n+d)/2, where d is the degree of f. We provide an asymptotic analysis for the quality of the bound in the regime where the order r is approximately equal to t*n for some scalar 0<t<1/2, showing that the relative error is in the order 1/2 - \sqrt{t(1-t)}. Our analysis relies on constructing suitable feasible solutions using polynomial kernels, which we obtain by exploiting symmetry and Fourier analysis on the Boolean cube. A crucial tool is relating the sum-of-squares hierarchy to another hierarchy of measure-based *upper* bounds (also introduced by Lasserre), and to exploit a link to extremal roots of orthogonal polynomials (in this case the Krawtchouk polynomials). Our error analysis in fact also applies to this second hierarchy.

This is based on joined work with Monique Laurent.